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Continuous-time state-space modelling of delinquent behaviour in adolescence and young adulthood

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Aim

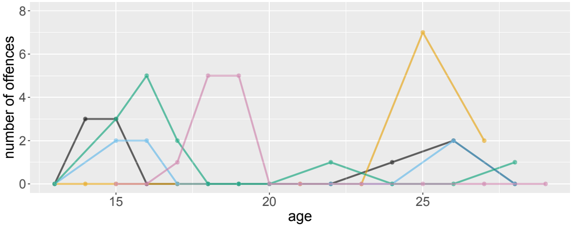
⇝ **investigate adolescents' delinquency over time**

Panel study *Crime in the Modern City*

- period: 2002-2017
- students in 7th grade (mostly 12–13 years old)
- repeatedly interviewed over 16 years (with 1–4 years between observations)
- self-administered questionnaires on various offences

⇝ variable of interest:
total number of offences

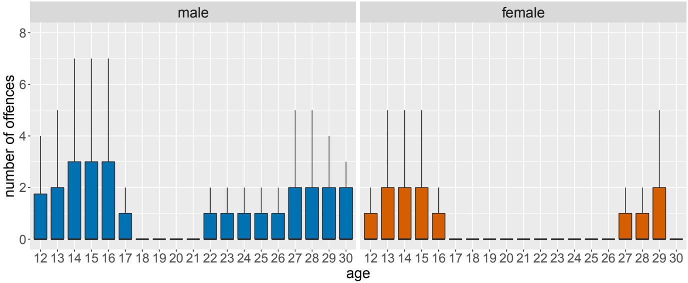
⇝ temporal persistence?!



Data

Inclusion of participants who committed ≥ 1 offences within study period:

- 12,327 observations from 1,093 adolescents (467 male & 626 female)
- 72.6% of observations: no delinquent behaviour
- 1–160 offences committed within 12 months (median: 3)



Data characteristics & modelling challenges

1. nonlinear relationship between age & number of offences
   $\rightsquigarrow$ model effect of age nonparametrically (using B-splines)

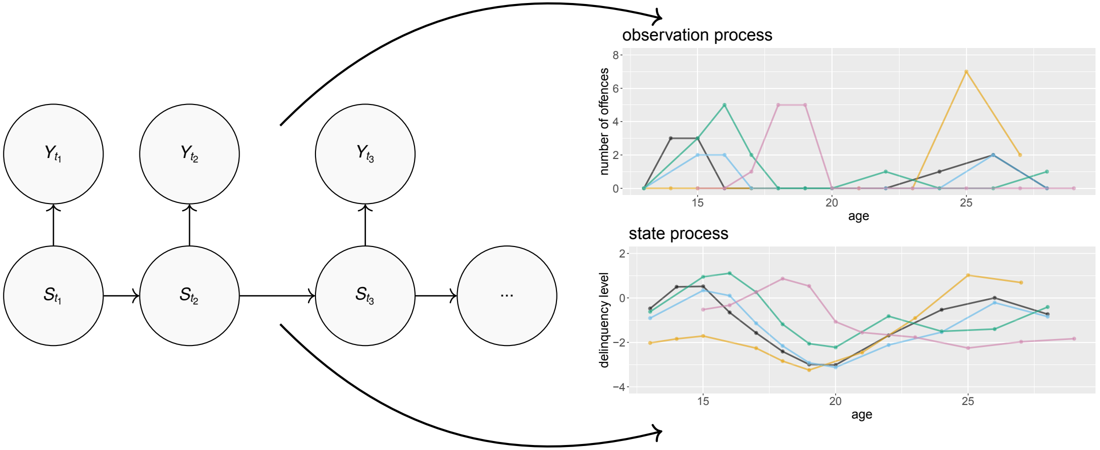
2. delinquency level is assumed to...
   - be a latent trait underlying the observed trajectories
   - change gradually over time

   $\rightsquigarrow$ use state-space model (SSM)

3. observations are irregularly spaced in time
   $\rightsquigarrow$ formulate model in continuous time

} continuous-time SSM

Basic model structure

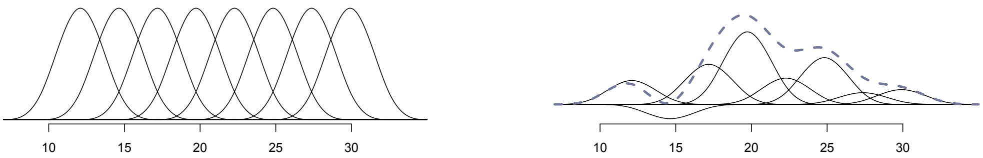


Model specification

Observation process:

$$Y_t \sim \text{NegBinom}(\nu_t, \phi),$$
$$\log(\nu_t) = S_t + f_1(\text{age}_t) + f_2(\text{age}_t) \cdot \text{gender}_t$$

with $f_i(\text{age}_t) = \sum_{l=1}^{8} \omega_{i,l} C_l(\text{age}_t)$



Model specification

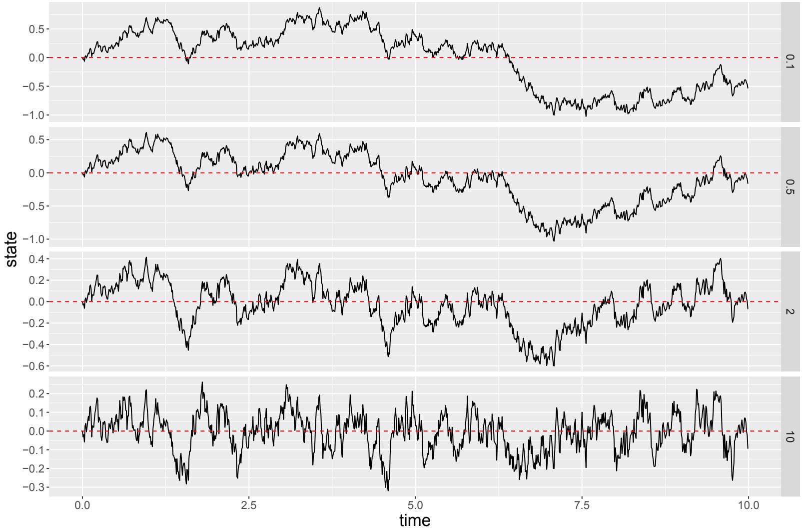
State process:

$$dS_t = \underbrace{-\beta S_t dt}_{\text{drift term}} + \underbrace{\sigma dB_t}_{\text{random fluctuations}}$$

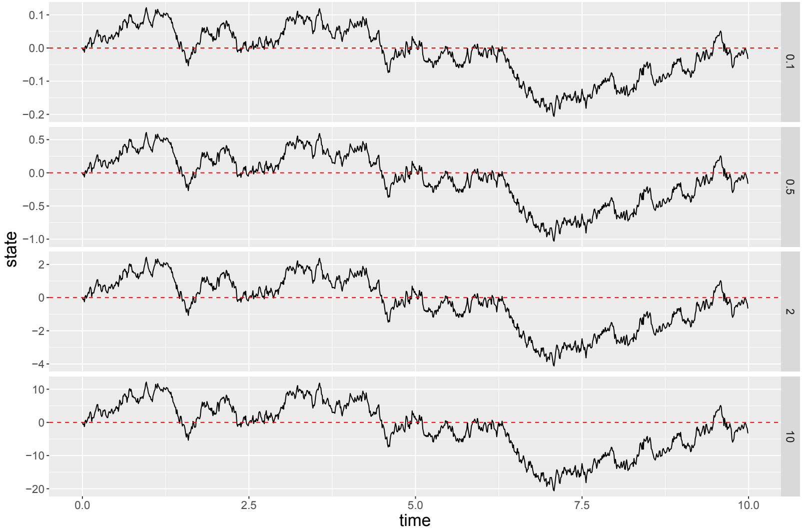
$\rightsquigarrow$ Ornstein-Uhlenbeck (OU) process
(continuous-time analogue of the AR(1) process)

$\rightsquigarrow$ delinquency level is persistent over time and changes gradually

OU process: varying $\beta > 0$



OU process: varying $\sigma > 0$



State process

OU process

- conditional distribution:

$$S_{t+\Delta t} | S_t = s \sim \mathcal{N}\left(\exp(-\beta \Delta t)s, \quad \frac{\sigma^2}{2\beta}(1 - \exp(-2\beta\Delta t))\right)$$

- limiting distribution:

$$S_t \sim \mathcal{N}\left(0, \frac{\sigma^2}{2\beta}\right)$$

Likelihood evaluation

$$\mathcal{L}_T = \int \ldots \int p(y_1, \ldots, y_T, s_1, \ldots, s_T) ds_T \ldots ds_1$$
$$= \int \ldots \int p(s_1) p(y_1 | s_1) \prod_{\tau=2}^{T} p_{\Delta_\tau}(s_\tau | s_{\tau-1}) p(y_\tau | s_\tau) ds_T \ldots ds_1$$

- intractable integration over all possible realisations of the state process at each observation time

⇝ **discretisation of state space** (Kitagawa, 1987):

- divide possible range $[b_0, b_m]$ into m intervals $B_i = (b_{i-1}, b_i)$, $i = 1, \ldots, m$ of length $(b_m - b_0)/m$
- let b_i^* denote the midpoint of B_i

$$\rightsquigarrow \quad \mathcal{L}_T \approx \sum_{i_0=1}^{m} \ldots \sum_{i_T=1}^{m} p(s_1 \in B_{i_0}) p(y_1 | s_1 = b_{i_0}^*) \prod_{\tau=2}^{T} p_{\Delta_\tau}(s_\tau \in B_{i_\tau} | s_{\tau-1} = b_{i_{\tau-1}}^*) p(y_\tau | s_\tau = b_{i_\tau}^*)$$

Likelihood evaluation

- approximation reframes SSM as an m-state hidden Markov model (HMM; Langrock, 2011)

$\rightsquigarrow$ HMM forward algorithm:

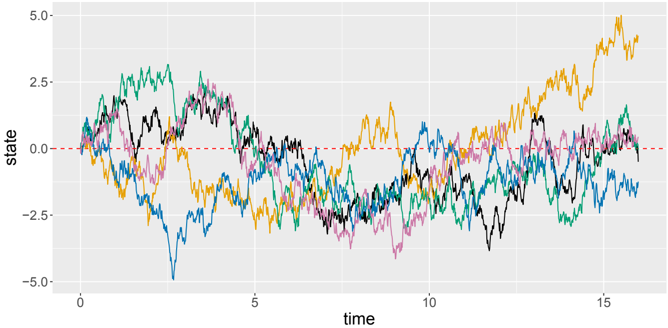
$$\mathcal{L} \approx \boldsymbol{\delta} \mathbf{P}(y_1) \Big(\prod_{\tau=2}^{T} \Gamma_{\Delta_\tau} \mathbf{P}(y_\tau) \Big) \mathbf{1}$$

- $\boldsymbol{\delta} = (\delta_1, \ldots, \delta_m)$ with $\delta_i = p(s_1 \in B_i)$ $\rightsquigarrow$ OU limiting distribution
- diagonal matrix $\mathbf{P}(y_\tau)$ with i-th entry $p(y_\tau | s_\tau = b_i^*)$ $\rightsquigarrow$ neg. binomial distribution
- transition probability matrix $\Gamma_{\Delta_\tau} = (\gamma_{ij}^{\Delta_\tau})$ with $\gamma_{ij}^{\Delta_\tau} = p_{\Delta_\tau}(s_\tau \in B_j | s_{\tau-1} = b_i^*)$ $\rightsquigarrow$ OU conditional distribution

Results: state process

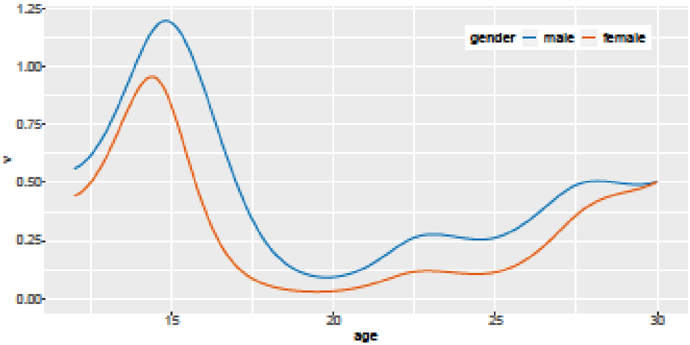
- estimated parameters of **OU process**: $\beta = 0.222, \sigma = 1.489$

$\rightsquigarrow$ limiting distribution: $S_\tau \sim \mathcal{N}(0, 2.23^2)$



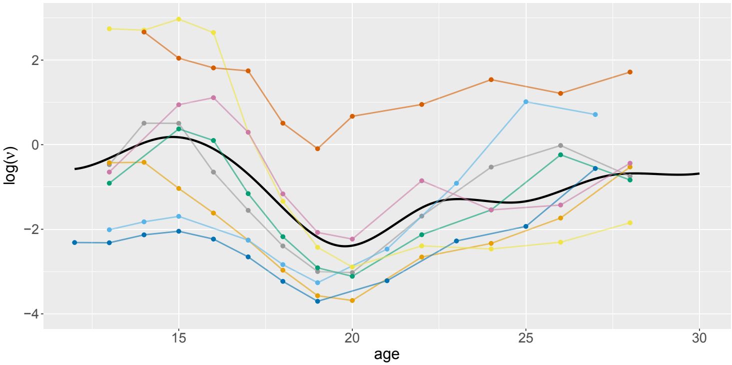
- model without underlying state process: ΔAIC : 1372

Results: covariate effects



- results overall correspond to the current state of research (e.g. Reinecke Weins, 2013)

Results: decoded states



- due to underlying delinquency levels, individuals' trajectories deviate from the overall age trend

Discussion

- **main result**:
  temporal persistence in the deviation of an individual's delinquency level from population mean

- (possible) **future work**:
  - inclusion of additional covariates
  - consider heterogeneity between adolescents

- **general continuous-time SSM framework**:
  - allows for non-Gaussian & non-linear specifications of the state & observation process
  - offers convenience of continuous-time HMM framework
  - suited for sequential data irregularly spaced in time

  $\rightsquigarrow$ great flexibility

References

- Kitagawa (1987), Non-Gaussian state-space modeling of nonstationary time series. *Journal of the American Statistical Association*.
- Langrock (2011), Some applications of nonlinear and non-Gaussian state-space modelling by means of hidden Markov models. *Journal of Applied Statistics*.
- Reinecke & Weins (2013), The development of delinquency during adolescence: a comparison of missing data techniques. *Quality & Quantity*.

Preprint: Mews et al. (2020), Maximum approximate likelihood estimation of general continuous-time state-space models, *arXiv*.

Thank you!