

Research Reports

Reactive and Proactive Control in Arithmetical Strategy Selection

Kerensa tiberghien*^a, Wim Notebaert^a, Bert De Smedt^b, Wim Fias^a

[a] Department of Experimental Psychology, Ghent University, Ghent, Belgium. [b] Parenting and Special Education Research Unit, KU Leuven, Leuven, Belgium.

Abstract

Individual differences in arithmetic have been explained by differences in cognitive processes and by arithmetic strategy use and selection. In the present study, we investigated the involvement of reactive and proactive control processes. We explored how variation in proactive and reactive control was related to individual differences in strategy selection. We correlated proactive and reactive measures obtained from the AX-CPT and an adjusted N-back task with a measure of strategy adaptiveness during a numerosity judgment task. The results showed that both measures of reactive control (of the AX-CPT and N-back task) correlated positively with strategy adaptiveness, while proactive control was not. This suggests that both cognitive control modes might have a different effect on adaptive strategy selection, where adaptive strategy selection seems to benefit from a transient (late) control mode, reactive control. We discuss these results in the light of the Dual Mechanisms Framework.

Keywords: proactive control, reactive control, numerical cognition, strategy selection

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*Corresponding author at: Department of Experimental Psychology, Ghent University, Henri Dunantlaan 2, 9000 Gent, Belgium. E-mail: Kerensa.tiberghien@UGent.be



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The last decade has witnessed an increased interest in individual differences in arithmetic (see Cappelletti & Fias, 2016 or De Smedt, Noël, Gilmore, & Ansari, 2013 for an overview). Generally, the selection and use of appropriate arithmetical strategies explain part of this variability (Imbo & Vandierendonck, 2007; Imbo, Vandierendonck, & Rosseel, 2007). In the present study, we investigated the involvement of reactive and proactive control processes in this selection of appropriate strategies. Recently, the involvement of cognitive control processes in arithmetic strategy use has been investigated (for a review, see Hinault & Lemaire, 2016) and sometimes interpreted as reflecting either reactive or proactive control. However, the specific involvement of these control processes was never explicitly investigated, which was the aim of the current study. Because strategy selection involves a decision-making process (i.e., choosing between the different strategies), cognitive control is involved to make an adaptive strategy selection. Investigating how proactive and reactive control are involved in the process of strategy selection, furthers our understanding on arithmetic strategy use.

To adequately perform mental arithmetic, a variety of cognitive processes are needed. Among these processes, we consider attention (e.g., focus on the arithmetic problem; Menon, 2010), working memory (e.g., holding and manipulating information in mind; Andersson, 2008; Raghubar, Barnes, & Hecht, 2010), response selection,

and executive functions. Miyake et al. (2000) identified three different functions in executive control and all three are known to contribute to individual differences in arithmetic: (a) shifting between tasks or mental sets (Yeniad, Malda, Mesman, van IJzendoorn, & Pieper, 2013), (b) information updating and monitoring of working memory representations (Raghubar, Barnes, & Hecht, 2010), and (c) inhibition of prepotent or dominant responses (Bull & Scerif, 2001; Gilmore et al., 2013; Kroesbergen, Van Luit, Van Lieshout, Van Loosbroek, & Van de Rijt, 2009; St Clair-Thompson & Gathercole, 2006).

Even relatively simple tasks such as mental arithmetic can be solved by means of different strategies. For instance, the solution to 7×9 can be directly retrieved from memory or can be calculated (e.g., $7 \times 9 = (7 \times 10) - 7$). Depending on the type of problem and on individual experience and skill, one or the other way is more efficient (Imbo & Vandierendonck, 2007; Imbo, Vandierendonck, & Rosseel, 2007). A strategy is defined as “a procedure or set of procedures to accomplish a high-level goal” (Lemaire & Reder, 1999, p. 365). To better understand where individual differences in performance come from, a thorough understanding of the use and efficiency of strategies is needed. Lemaire and Siegler (1995) proposed four dimensions of strategic competence on which individuals can differ: 1) *strategy repertoire* or the available strategies for that task are used to accomplish the task, 2) *strategy distribution* or the frequency at which the strategies are used, 3) *strategy efficiency* or how fast and accurate each strategy is used and 4) *strategy adaptiveness / strategy selection* or how adaptively strategies are selected on a trial-by-trial basis. Strategy adaptiveness can be measured by looking at how participants are able to alter their strategy choices to task parameters and is analyzed by how often the most efficient strategy (that is, the strategy that ensures the highest probability of an accurate and fast performance) was selected on each item. This means the participant needs to make systematic strategy choices on a trial-by-trial basis, which requires the participant to flexibly switch between different strategies. We argue that this selection process critically depends on cognitive control processes, as we have tested in the current study.

In general, previous studies have tried to explain individual differences in arithmetic by investigating the impact of specific executive functions thereby trying to identify which executive functions (i.e., inhibition, updating and switching) are involved in the different aspects of arithmetic, such as numerosity discrimination (Fuhs & McNeil, 2013) and mental arithmetic (Bull & Scerif, 2001). Given the involvement of multiple processes in arithmetic, planning and control processes are required to organize the execution of all operations required for arithmetic. This is usually referred to as cognitive control. Rather than focusing on specific executive functions, we have turned our attention to these supervisory planning and control processes that govern these specific functions. Because strategy selection entails a decision-making process (i.e., choosing between the different strategies), we contend that cognitive control must be involved in order to make an adaptive strategy selection. Also, Botvinick (2007) describes that decision-making and conflict monitoring go together. More precisely, Botvinick argues that conflict monitoring drives a form of avoidance learning, which biases behavioral decision-making toward cognitively efficient tasks and strategies. Because strategy selection also requires decision-making, though no form of conflict monitoring is needed, we can assume that cognitive control is also required for adaptive strategy selection.

The involvement of such cognitive control processes in arithmetic strategy selection and execution has begun to be addressed empirically (Hinault, Lemaire, & Phillips, 2016; Hodzick & Lemaire, 2011; Taillan et al., 2015; for a review, see Hinault & Lemaire, 2016). For example, Taillan and colleagues (2015) investigated the neural correlates of strategy selection in a computational estimation task. The results showed greater brain activations

in the right anterior cingulate cortex (ACC), dorsolateral prefrontal cortex (DLPFC) and angular gyrus (AG) when selecting the better strategy on each problem. Taillan et al. believe the AG reflects the participants' use of a heuristic that helps in selecting the better strategy in the computational estimation task. Also, the ACC and DLPFC were previously observed in conflict tasks (e.g., Botvinick et al., 2001, 2004; Braver, 2012; Braver et al., 2009; Kerns, 2004, 2006), the authors therefore interpreted the results as reflecting response conflict detection and resolution and suggested that executive control processes are necessary during strategy selection. Another study has shown that cognitive control is influenced by sequential modulations of arithmetic strategies (Hinault, Badier, Baillet, & Lemaire, 2017). The results of this study have demonstrated that the efficiency of strategy selection and execution differs depending on the previous item. If the previous item contained interference, the strategy selection and execution on the current item was more efficient. Hinault et al. interpreted this finding as reflecting proactive preparation from one trial to the next. Even though the findings of sequential modulations were interpreted in terms of cognitive control processes, however, the specific involvement of these cognitive control processes (i.e., proactive and reactive control) was never explicitly investigated. For that reason, our study wanted to explore how proactive and reactive control are involved in arithmetic strategy use.

Cognitive control helps us to make appropriate adjustments in perceptual selection, response biasing and online maintenance of contextual information (Botvinick et al., 2001). Of course, cognitive control is also susceptible to variability and individual differences. To better understand the diversity of cognitive control, Braver, Gray, and Burgess (2007) have proposed the Dual Mechanisms of Control (DMC) framework. The DMC framework explains the individual variability in cognitive control in terms of temporal dynamics of these control processes. The DMC framework posits two control modes, a proactive (early) and a reactive (late) mode. *Proactive control* involves sustained active maintenance of all task goals (i.e., task instructions, identity of previous trials/ stimuli, cues for later behavior, etc.), which allows rapid and efficient responses. Proactive control generally occurs before the onset of a stimulus. Proactive control allows an appropriate response to cognitively demanding events, such as highly interfering stimuli. On the other hand, *reactive control* works as a 'late-correction' mechanism, that is the reactivation of task goals in a transient manner. Reactive control occurs after the stimulus onset. Reactive control mechanisms are only engaged when needed, which requires the stimulus to be sufficiently salient in order to reactivate the task goals. According to the DMC framework, both proactive and reactive modes are assumed to lead to an adequate performance on a specific trial. The advantage of proactive control is that both the planning and behavior can be altered continuously throughout the task to facilitate optimal performance before the stimulus onset, whereas reactive control results in a certain late intervention, that allows skills, procedures and habits to be learned while still having the possibility to override them if needed in a late manner.

Although it has, to our knowledge, never been explicitly investigated, one could in principle predict that both cognitive control modes, proactive and reactive control, help in the selection of the most adaptive strategy. Because proactive control aids in maintaining task instructions and type of stimuli in mind, it seems logical that it helps in knowing which strategy to select on which type of stimuli (based on prior knowledge). That is, proactive control helps in preparing attention towards item characteristics and strategy features to select a strategy, before the stimulus is presented. This proactive control preparation can also be influenced by expectations of the participant, a personal preference for a certain strategy or even the previous trial. Reactive control, on the other hand, aids in situations in which anticipation or preparation of the characteristics of the upcoming stimulus is not the most efficient way of performing the task. In that case, reactivation of required

information is triggered in a transient manner (upon seeing the stimulus) and therefore acts as a late correction mechanism. We therefore predicted that both cognitive control modes would correlate with individual differences in adaptive strategy selection in arithmetic.

The Present Study

In our study, we wanted to understand to which extent variability in strategy selection is related to individual differences in proactive and reactive control. For this purpose, we obtained measures for proactive and reactive control by means of the classical paradigms (AX-CPT, Braver et al., 2001; modified N-back, Marklund & Persson, 2012) and looked at how they correlated with strategy adaptivity in a numerical cognition task. Concretely, we selected two tasks to measure both cognitive control modes, that is proactive and reactive control. The first task we used is an altered version of the well-known AX-CPT task (Braver et al., 2001; Paxton et al., 2008). This task is a continuous performance task (CPT) in which the participant sees a continuous stream of letters presented in cue-probe pairs. The participant is asked to respond when seeing an X (probe), but only if it is preceded by an A (cue). In this task, proactive control helps when the cue is informative. When the cue is not A, a non-target response is prepared. Reactive control helps when the cue is uninformative (when the cue is A), then it is appropriate to wait for the probe to make the correct response. The second task we used is designed by Marklund and Persson (2012). It is a *modified 3-back paradigm*. Marklund and Persson designed the task to have a more cognitively challenging task than the AX-CPT. The task consists of two parts: A conventional 3-back task (uncued condition), that emphasizes reactive control and an adapted version (cued condition) with embedded cues that predict high-interference trials. In both parts of the 3-back task, lures (2-back or 4-back) were used to increase difficulty. In the conventional task these lures were uncued and therefore the participant could only react at the time of occurrence of the lure, for which good reactive control is needed. In the cued version, the lure is cued (by the number 2 or 4, meaning that the current stimulus will reappear in respectively 2 or 4 trials). This contextual information needed to be maintained over trials, which should encourage the use of proactive control. The main reason for using two different tasks to measure proactive and reactive control is because we wanted to see whether both tasks would measure the cognitive control modes in the same way, which to our knowledge has not been investigated before. Furthermore, the use of different tasks allowed us to see if the effects of the cognitive control modes on strategy adaptiveness were independent of the task used, especially since both tasks carry another difficulty level, where N-back is the more cognitively challenging task.

Lemaire and colleagues (Lemaire, Arnaud, & Lecacheur, 2004; Lemaire, Lecacheur, & Farioli, 2000; Siegler & Lemaire, 1997) introduced a choice/no-choice paradigm in which participants were asked to round multiplications either up or down depending on the item itself (e.g., 78×34 is an example of a rounding up problem (80×40), while 71×36 is an example of a rounding down problem (70×30)). With this paradigm, they have found a way to look at all four strategy dimensions (strategy repertoire, strategy distribution, strategy efficiency and strategy adaptiveness). The measure of adaptivity in this task checked if having a choice among different strategies also resulted in faster performance on the task as a whole (in the Choice condition). The reasoning is that if participants can freely choose between different strategies and they make this strategy choice adaptively, then they would choose the most efficient strategy on each trial. Despite the enlightening aspects of this parameter, however, the measure of adaptivity used by Lemaire is not optimally suited to investigate individual or group differences because it only looks at information from the Choice condition, which would result in biased estimates due to strategy selection. That is, strategy selection effects are caused by an

unequal use of the available strategies. For that reason, [Luwel and colleagues \(2003\)](#) have developed a novel measure of adaptivity, where they used unbiased estimates (i.e., estimates that are free of strategy selection effects) of strategy performance. This unbiased measure of adaptivity, is based on the location of change points, that is the point at which the participant switched from one strategy to another. Because we argue that cognitive control is needed for adaptive strategy selection, the parameter based on the change point seems best suited for our research purpose, because changing adaptively from one strategy for another requires a certain amount of cognitive control. This novel strategy adaptiveness parameter was first tested in a numerical judgment task ([Luwel et al., 2003](#)). The numerical judgment task is a choice/no-choice paradigm in which participants see a 7 x 7 grid that consists of a given number of colored and empty squares. Participants have to indicate how many squares are colored. If there are more empty squares than colored squares, the most efficient strategy is 'addition' (i.e., count the colored squares), but if there are more colored squares than empty squares, the most efficient strategy is 'subtraction' (i.e., count the empty squares and subtract them from the total number of squares, which remains fixed throughout the experiment). The task can only be solved by using these two dominant strategies, that can also be easily checked by the experimenter, because the participant is required to manually point to the squares in the grid (e.g., either the colored or the empty squares) while counting.

Because the numerosity judgment task requires the discrimination between the number of colored and empty squares to select a strategy, it can be predicted that the ability to discriminate numerosities ([Halberda, Mazocco, & Feigenson, 2008](#)) influences the performance of a participant. That is, if a participant performs well in discriminating different magnitudes, the participant might select a strategy in a more efficient way (e.g., faster) than a participant who does not perform as well on magnitude discrimination. To check the possibility that strategy selection is related to variability in magnitude discrimination, we additionally included a non-symbolic number comparison task, more specifically the Panamath ([Halberda, Mazocco, & Feigenson, 2008](#)). This task is designed to measure approximate number system (ANS) acuity, which refers to the ability to discriminate between different magnitudes.

When considering the influence of the cognitive control modes on strategy adaptiveness, we predicted that proactive control helps to keep in mind the different strategies themselves (i.e., addition and subtraction) and knowledge on when which strategy needs to be selected to perform adaptively (i.e., with more empty squares, best to use addition; with more colored squares, best to use subtraction). As mentioned before, proactive control might also involve influences of expectations of the participant, a personal preference for a certain strategy or even the previous trial. Reactive control helps the participant on a trial-by-trial basis when preparation of the upcoming stimulus is not the most efficient way to perform the task, for example by reactivating the latent rules about the strategies (i.e., when there are more colored squares, it is best to use subtraction, while more empty squares would require addition) when there are about 50-50% colored and empty squares. To summarize, we hypothesized that participants who perform better on the proactive and/or reactive measures, will also perform better at selecting the most adaptive strategy in the numerosity judgment task.

Method

Participants

Forty-five psychology students from Ghent University participated in the experiment (39 females, mean age = 19, range = 18 - 21; 42 right-handed). All participants received a course credit for participation. Participant raw data of the present work is available at DANS (Data Archiving and Networked Service), see [Supplementary Materials section](#).

Tasks

The participants performed a battery of tasks but only the tasks relevant for this study are described: a numerosity judgment task in a choice/no-choice paradigm, an AX version of the continuous performance task (CPT), a N-back task and the Panamath. All tasks, except the Panamath (Halberda, Mazocco, & Feigenson, 2008), were controlled by E-Prime (Psychology Software Tools, Pittsburg, PA) and displayed on a 1600 x 900-resolution screen. The computer was placed on average 50 cm in front of the participant.

Numerosity Judgment Task (Choice/No-Choice Paradigm)

The numerosity judgment task is an easy numerical task in which the participant is required to count a certain number of colored squares in a 7 x 7 grid. Depending on the number of colored squares, different strategies can be used (e.g., addition or subtraction). The task is presented as a choice/no-choice paradigm, which makes it possible to examine all four dimensions of strategy use.

The choice/no-choice task we used is the one by Luwel, Verschaffel, Onghena, and De Corte (2003). In this experiment participants saw a square grid of 7 x 7. Each small square had the size of 1 x 1 cm. The outline of the grid including the intersections was colored red, while the squares themselves could be green (colored) or black (empty). In each condition, the participant was shown 49 trials (i.e., one trial of each numerosity of colored blocks). Within each condition, the trials and placement of the colored blocks in the grid were randomized. The participant was asked to give the number of colored blocks on each trial (independent of the condition). Depending on the number of colored (green) blocks, a different strategy can be used. When there were more empty blocks compared to colored blocks, the most efficient strategy is to count the colored blocks, thus the addition strategy. When there were more colored blocks than empty blocks, the most efficient strategy would be to count the empty blocks and subtract them from the total grid (49 blocks) in order to gain the correct number of colored blocks, thus the subtraction strategy (see [Figure 1](#)).

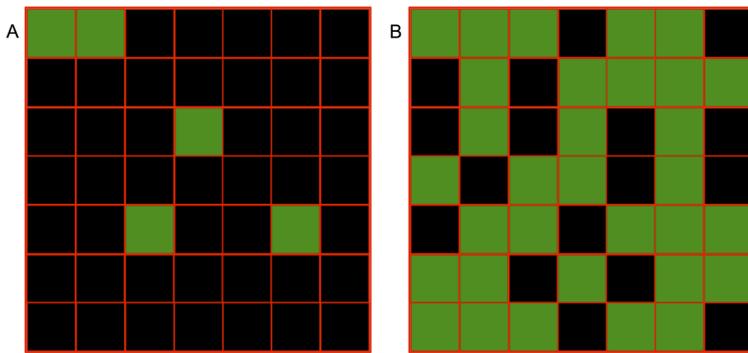


Figure 1. Examples of stimuli in the numerical judgment task.

Note. A trial that elicits addition (A) and a trial that elicits subtraction (B).

Choice/No-choice conditions — Each participant performed three conditions: choice, no-choice addition (forced addition, FA) and no-choice subtraction (forced subtraction, FS). In the choice condition, participants could choose freely between the two strategies to count the numerosity of the colored blocks in the grid. In the FA condition participants were asked to determine the numerosity only by means of the addition strategy, which means the participants were not allowed to use the subtraction strategy. In the FS condition participants were asked to determine the numerosity by using the subtraction strategy only.

All participants started with the choice condition to exclude any influence of strategy choices based on recency effects. The no-choice conditions were counterbalanced over participants. The experiment started with 5 practice trials (3, 14, 22, 31 & 46 colored blocks), which represented all sorts of numerosities from the continuum. After the practice, participants performed the three conditions, each containing 49 trials. Because the participants already used both strategies in the practice block, participants were familiar with both available strategies (addition and subtraction). Before the start of each condition, the participant was told which strategies were allowed in this block. The participant was asked to determine the numerosity of the colored (green) blocks. To check which strategy the participant used, they were asked to point out the squares they were counting on each trial. This also provided an extra check to make sure the participant used the required strategy in the no-choice conditions. The stimuli remained on the screen until the participant answered and the experimenter (one experimenter collected all the data) pressed ‘Enter’ immediately which recorded a reaction time. The experimenter then typed the answer and which strategy was used by the participant. The used strategy was derived from the pointing behavior of the participant and if the experimenter was not entirely sure which strategy the participant used, the participant was asked explicitly to state the used strategy on that trial.

Strategy adaptiveness measure — In the numerosity judgment task we calculated a strategy adaptiveness measure, which was our dependent variable. The measure we used is the same as in [Luwel, Verschaffel, Onghena, and De Corte \(2003\)](#) and is based on the location of change points. By change point we mean the point at which the participant switched from an addition strategy to a subtraction strategy in the Choice condition. Because we had information on the participant’s strategy by their pointing behavior, it was possible to identify the individual change points.

The strategy adaptiveness measure is the difference between the actual change point and the projected change point (see [Figure 2](#)). The *actual change point* is calculated by applying a piece-wise regression model

to the individual's reaction time data of the Choice condition. The point at which the slope changes, thus the breakpoint of the regression lines, is the actual change point.

The *projected change point* is derived from the reaction time data in the no-choice conditions. A first step in calculating the projected change point is looking at the individual reaction time patterns to find the break point of the regression line of each of the no-choice conditions, either at the end (FA condition) or at the beginning (FS condition) of the regression line. In the FA condition, this meant the participant has used addition for mostly all numerosities, except when there were many colored squares, then the participant might have started to use another strategy (counting by full rows or in small groups of blocks or even unconsciously using subtraction). This phenomenon also occurred in the FS condition: When there were many empty squares, participants tended to use the same strategies as mentioned above. In the cases where a strategy other than pure addition or subtraction was used (e.g., trials at which the participants were that fast they did not even point out which squares they were counting), the participants were not explicitly asked which strategy was used, because these trials would be excluded from further analyses (because the reaction time would be much lower on these trials, they would impair the linear regression of that no-choice condition, as explained in the next paragraph).

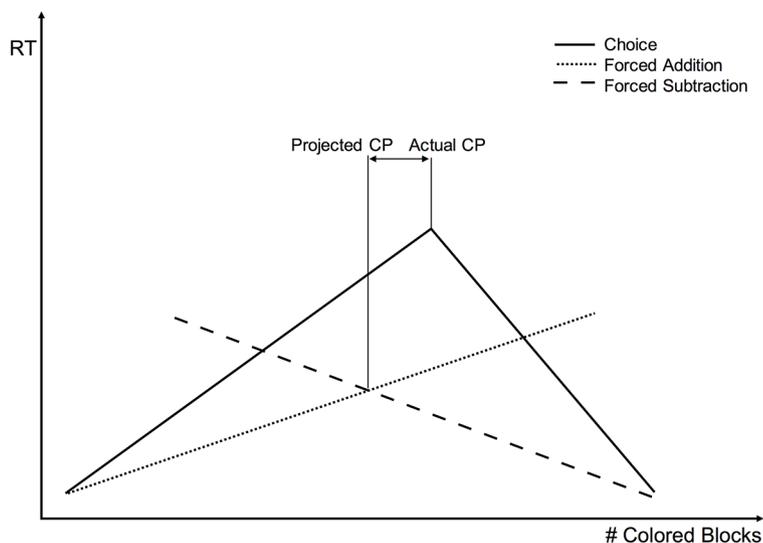


Figure 2. The calculation of the strategy adaptiveness measure.

Note. RT = reaction time. CP = change point. The projected change point is the intersection of the regression lines of the Forced Addition condition and Forced Subtraction condition. The actual change point is the breakpoint of the Choice condition regression line. The strategy adaptiveness measure is the difference between the actual change point and the projected change point.

First, break points of the regression line were calculated by means of a piece-wise regression on the individual's response time patterns of both no-choice conditions. The data before (in FS condition) and the data after (in FA condition) the break point were excluded from the data patterns, because they would impair the linear regression. Second, influential outliers were excluded for each participant by means of a Cook's D statistic (Myers, 1990; Neter, Kutner, Nachtsheim, & Wasserman, 1996). Third, simple linear regressions were performed on the individual reaction time data of both no-choice conditions. Finally, regression lines of the computed regression equations of both no-choice conditions were plotted in a single graph. The intersection of both regression lines was the projected change point.

The projected change point indicated the trial on which it is most efficient to switch from the addition strategy to the subtraction strategy, for each participant individually. If, in the choice condition, that individual switched to the subtraction strategy on that specific trial (i.e., the actual change point), this meant that the individual is perfectly adaptive. Therefore, the measure of adaptivity is the difference between the actual and the projected change point. Thus, the smaller this difference, the more adaptive the individual's strategy selection.

AX-CPT

The AX-CPT we used, is a slightly altered version of the task by Braver and colleagues (2001). The task obtained measures of proactive and reactive control (see Figure 3).

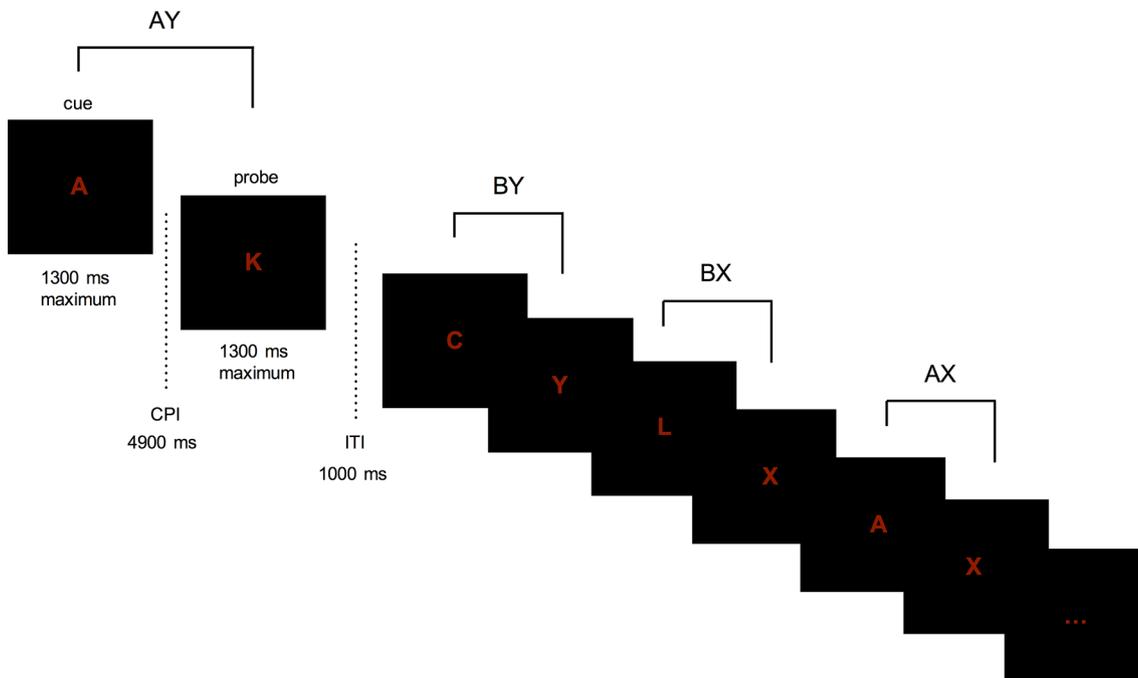


Figure 3. Trial sequence of the AX-CPT task.

Note. CPI = cue-probe interval. ITI = interstimulus interval. AX = cue A and probe X. AY = cue A and probe Y. BX = cue B and probe X. BY = cue B and probe Y.

The participants saw letters presented sequentially. Each trial sequence consisted of a cue and a probe. There were two types of trials: target trials and non-target trials. Target trials were those in which the cue was an A and the probe an X (called AX-trials). Non-target trials contained: 1) AY-trials in which the cue was an A (valid cue), but the probe was any other letter than X (invalid probe), 2) BX-trials in which the cue was any other letter than A or X (invalid cue), and the probe was an X (valid probe), and 3) BY-trials in which both cue and probe were any other letter than A or X (invalid cue and probe). The letters K and Y were excluded because their appearance is quite similar with the letter X. Target-trials had a frequency of 70% and non-target trials 30% (10% for each type). Stimuli (both cues and probes) were presented in the center of the screen for 300 ms, in red on a black background in 18-point Courier New. The cue and probe remained on the screen until the participant responded with a maximum of 1300 ms. The interval between cue and probe lasted for 4900 ms and the intertrial interval was 1000 ms. The participant was asked to press a button every time a non-target appeared (e.g., response with index finger), but when the target trial appeared (a probe with the letter X but

only when preceded by the cue A), the participant needed to press the other side (e.g., response with middle finger). This meant that every cue was also seen as a non-target. Responses were given with the dominant hand and measured with a Response Box. Response mappings were counterbalanced over participants.

We informed the participants that letters would be presented sequentially and that they had to press a button for each letter presented. For example, they were instructed to press with their index finger for any letter presented and with their middle finger only if the current letter was an X and when it was preceded by an A (so when they saw an X not preceded by an A, they still had to use their index finger). The participants started with a practice phase containing 6 trials (3 AX-trials, 1 BX-trial, 1 BY-trial and 1 AY-trial) to get familiar with the task. Afterwards, there were two blocks of 40 trials (28 target-trials and 12 non-target trials) with a small break in between.

Proactive / Reactive control measures — In the AX-CPT task, it was necessary to maintain a mental representation of the context information and update this information on a trial-by-trial basis. Target trials (AX) were highly frequent (occurrence of 70%) in the task. When the participant made a mistake, two response bias errors were possible: 1) *perceptual/intuitive bias errors*, which occurs when the participant failed to inhibit the dominant response, that is the target response (AX). These errors occur on BX-trials. Upon seeing the probe X, the participant failed to inhibit the target (dominant) response, while a non-target response was required. 2) *expectancy bias errors*, which occurred when the cue is an invalid predictor to the probe, that is on AY-trials. The cue A directed attention to the dominant response (AX), resulting in a false alarm (Braver et al., 2005; Iselin & DeCoster, 2009).

BX-trials were trials in which *proactive control* can help improve the participant's performance (Braver et al., 2005). When a cue B was shown, there were two possibilities for the probe. The probe could either be 1) X (BX-trial) or 2) Y (BY-trial), which both required non-target responses. Therefore, by seeing cue B, the participant could already prepare for this non-target response. If the participant had a good proactive control mechanism, this meant the performance on these BX-trials will be good. Proactive control was measured by this formula: $\text{Errors [BX / (BX + overall)]}$. The lower the measure, the better the participants' performance on proactive trials.

AY-trials were trials to measure *reactive control*. When cue A is shown, there were again two possibilities for the probe. The probe could either be 1) X (AX-trial), which is the target trial or 2) or Y (AY-trial), which is a non-target trial. This meant that when cue A is shown, the participant had no information about which type of trial this is and needed to wait for the probe to appear to make the correct response. Since the participant was required to make a last-minute decision (based on the probe), active preparation was not possible and therefore reactive control was used to perform the task adequately. The formula for the reactive control measure is: $\text{Errors [AY / (AY + overall)]}$. The lower the measure, the better the performance on reactive trials.

N-back

The N-back task we used, more specifically the 3-back task, was based on the task by Marklund and Persson (2012). A standard N-back task is designed to measure maintenance and updating processes of working memory. This version of the N-back is designed to obtain measures of proactive and reactive control (see Figure 4).

The participants saw letters presented sequentially. The participant's task was to determine whether the current letter is a target (the same as 3 letters before) or a non-target (not the same as 3 letters before). The task consisted of two conditions: cued and uncued. The experiment contained two uncued blocks and two cued blocks, each consisting of 32 trials. The 32 trials contained 8 target (3-back) trials and 24 non-target trials. The non-target trials were divided in control-trials (an entirely new letter) or lures. Lures were trials in which the current letter has been shown 2 or 4 letters before, we therefore only had 2-back lures and 4-back lures. These trials increased the task difficulty and made sure the participant was not able to detect the target purely on the basis of recency effects. Stimuli were centrally presented letters with a number above them in white on a black background in 18-point Courier New. The stimulus appeared for 2000 ms followed by an interstimulus interval of 1000 ms. The participant was given 3000 ms to respond.

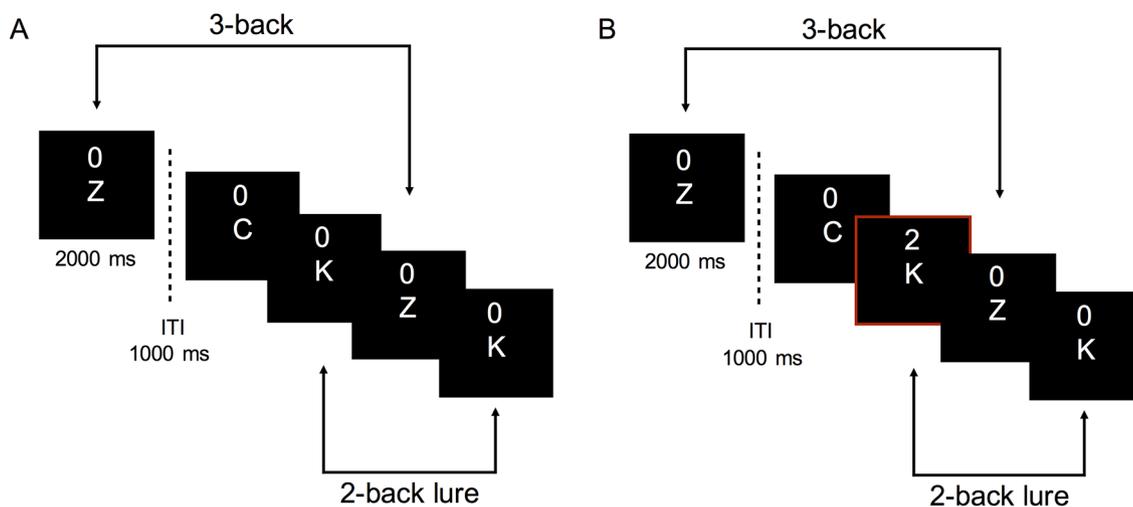


Figure 4. Trial sequence of the 3-back task for both conditions.

Note. ITI = intertrial interval. In the uncued condition, only the number '0' was shown above the letters (A). In the cued condition, the numbers '2' or '4' were shown above the letters to indicate a 2-back or 4-back lure, respectively (B).

The 3-back task had two conditions. The first condition was the uncued, regular condition. In this condition, the participant saw a number above the letter which would always be 0. Therefore, this number was not informative and is called uncued. In the second, cued condition, the participant could see a 0, 2 or 4 above the letter. A 0 was again uninformative, but a 2 or 4 meant that the same letter being presented at that time, would come back 2 or 4 letters later, respectively. This meant that the participant gained information about the lures based on this number, the cue.

We informed the participant that letters would be presented sequentially and they had to respond for each letter presented. For example, the participant was instructed to press with the index finger for a target (3-back) and with the middle finger for a non-target. At the start of the experiment we did not inform participants of the number 0 above the letter. The participant started with 10 practice trials, followed by the two uncued blocks. After these blocks, the participant was given instructions about the cue: "Starting from now, the number you saw above each letter might become meaningful. You might see a number 0, 2 or 4. When you see the number 0, this is not informative, but when you see a 2 or a 4 this means that the same letter you are seeing now will come back in 2 or 4 letters, respectively. This might help you in discovering non-targets (2-back and 4-back).

Keep in mind the task remains the same, you still need to detect the targets (3-back).” After these instructions, the participant performed the two cued blocks.

Proactive / Reactive Control Measures — In the N-back task, lures are the most informative trials for our experimental purpose, this is to have a measure of proactive and reactive control. In the uncued condition, the participant could not anticipate on an upcoming lure, therefore only reactive control was activated when performing a lure-trial. The *reactive control* measure of this task was obtained by this formula: Reaction time [$\text{lure}_{\text{uncued}} / (\text{lure}_{\text{uncued}} + \text{control}_{\text{uncued}})$]. A lower measure means more (adequate) use of reactive control.

In the cued condition, lure trials were accompanied by a predictive cue (the first time you saw a letter which would be a lure, you were informed whether this letter would come back in 2 or 4 trials, the lure itself). This cue might have helped the participant to increase performance on these high-interference trials, although it was necessary to maintain this additive information. Participants were expected to increase performance when using proactive control on these cued lures. *Proactive control* was measured by means of this formula: Reaction time [$\text{lure}_{\text{cued}} / (\text{lure}_{\text{cued}} + \text{control}_{\text{cued}})$]. A lower measure means more (adequate) use of proactive control.

Panamath

Panamath is a task designed to measure ANS (approximate number system) acuity (Halberda, Mazocco, & Feigenson, 2008; <http://www.panamath.org>). ANS acuity refers to a participant’s ability to discriminate between different magnitudes. The value we extracted as a measure for approximate number system was the Weber fraction. Weber’s law implies that the difference between the two stimuli will be perceived more easily as the ratio between the magnitudes of the two stimuli increases. A smaller Weber fraction indicates a better discrimination ability, this means the ANS representation obtains less noise.

Task stimuli consisted of a group of yellow dots on the left side and blue dots on the right side, with 100% separation, on a grey background. There were four ratio bins: 1.11; 1.20; 1.35 and 2.31. The numbers of dots ranged from 5 to 21 and the average dot had 36 pixels, with a 20% variation in diameter. Based on these settings, 8 combinations were possible, which were displayed 10 times each. The first trial started with a fixation cross and the participants were asked to press ‘Space’ to start the experiment. The other trials started by presenting the stimulus for 706 ms. After the stimulus, a backward mask was presented for 200 ms, consisting of colored pixel noise followed by a 2000 ms interstimulus interval. Participants were required to judge which group had more dots (yellow or blue) and hit the corresponding key on the keyboard (F = left; J = right).

The participant was told two groups of dots would appear on the screen (one on the left and one on the right in a certain color). The task was to detect the group which contained more dots and press the corresponding key on the keyboard (F = left; J = right). The task started with 2 practice trials followed by 80 experimental trials. We informed the participant that there would be no visual or auditory feedback on each trial. A screen with performance statistics was shown at the end of the experiment.

General Procedure

Each participant was tested individually, during sessions that lasted for maximum an hour. In the first session, the participants performed the AX-CPT task, the N-back task and the Panamath. These tasks were

counterbalanced across participants. Approximately two months later, we tested the participants for an hour. In that hour, the participants performed the numerosity judgment task.

Results

Two participants were excluded from the analyses. One participant performed under chance level on both the AX-CPT and N-back task. The other participant was excluded due to a failure of the Panamath software. Descriptive statistics of all tasks can be found in Table 1. For all analyses, estimates of effect size were reported: partial eta-squared was reported for the ANOVA (0.01 = small, 0.06 = medium, 0.14 = large effect), and Cohen's D (0.2 = small, 0.5 = medium, 0.8 = large effect) for t-tests (Cohen, 1988).

Table 1

Descriptive Statistics of the Experimental Tasks

Measure	N	M	95% CI
NJT: Strategy Adaptiveness	43	2.12	[1.52 - 2.72]
AX-CPT: Proactive Control	43	.57	[.42 - .72]
AX-CPT: Reactive Control	43	.27	[.15 - .39]
N-back: Proactive Control	43	.52	[.51 - .54]
N-back: Reactive Control	43	.56	[.55 - .57]
Weber Fraction	43	.18	[.15 - .20]

Note. NJT = numerical judgment task. CI = confidence interval.

Before discussing the relation between strategy adaptivity and cognitive control, general findings of the numerical judgment task are presented. First, the reaction times and error rates were calculated for each condition, which can be found in Table 2. Next, we have performed a one-way ANOVA with repeated measures on the reaction times and error rates, with Condition (Choice, FA, FS) as a within-subject measure. Results showed that there is a significant difference in reaction time depending on Condition ($F(2,84) = 254.50, p < .001, \eta_p^2 = .86$). Post-hoc analyses revealed that trials in the Choice condition ($M = 6201, SD = 1024$) were solved faster than trials in the FA condition ($M = 8764, SD = 1805; t(42) = -13.78, p < .001, d = -2.10$) and the FS condition ($M = 10654, SD = 2120; t(42) = -19.04, p < .001, d = -2.9$). Additionally, trials were solved faster in the FA condition compared to the FS condition ($t(42) = -11.20, p < .001, d = -1.71$). The fact that trials in the Choice condition were solved faster than the no-choice conditions is a first indication of adaptive strategy selection. Results on the error rates showed no significant effects for Condition (Choice: $M = 13\%, SD = 9\%$; FA: $M = 12\%, SD = 9\%$; FS: $M = 15\%, SD = 11\%$; $F(2,84) = 2.47, p > .09$).

Table 2

Reaction Times (in Milliseconds) and Error Rates (%) of the Numerosity Judgment Task.

Behavioral Measure	Condition		
	Choice	FA	FS
Reaction time	6201 (1024)	8764 (1805)	10654 (2120)
Error rate	13% (9%)	12% (9%)	15% (11%)

Note. FA = forced addition. FS = forced subtraction. Standard deviations are in parentheses.

Because the reaction time (RT) data significantly differed across conditions, we have depicted the data in Figure 5. The regression of the Choice condition nicely follows that of the FA for the small numerosities and that of the FS condition large numerosities, which shows that participants were adaptive in choosing their strategy in the Choice condition. The figure nicely reveals that the RTs are longer for the No-Choice conditions once the actual change point was passed. The figure additionally shows that the average change points were not exactly in the middle (i.e., 24-25 colored blocks), which is because in the middle range participants tend to use addition more often than subtraction. The classic analysis of adaptivity that only considers the Choice condition is only a first step to look at strategy adaptiveness. More compelling data on individual differences of strategy adaptiveness are revealed via the measure we have used (Luwel et al., 2003), which takes into account the point at which it is most adaptive to switch from addition to subtraction. Note that all of these data points in Figure 5 are average RTs across participants, thus the individual RTs, change points and number of excluded trials differed per participant.

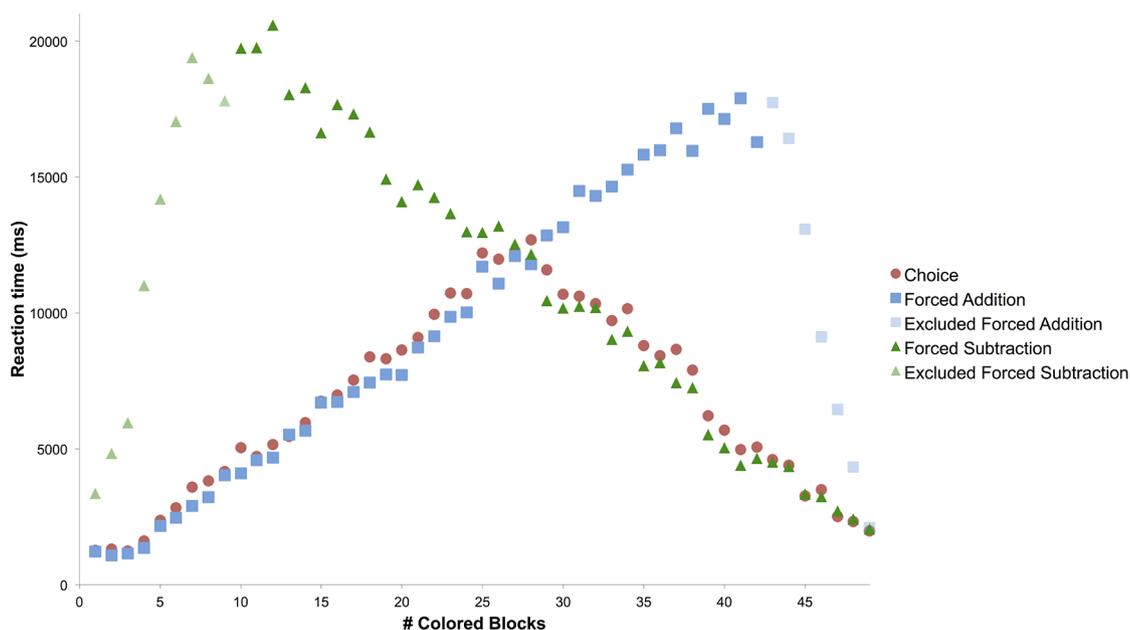


Figure 5. Reaction times (ms) of the numerosity judgment task averaged across participants per condition.

Note. The greyed-out colors represent the average excluded trials of the no-choice conditions. Note that the average reaction times are depicted on this figure, while the individual RTs, change points and number of excluded trials differed per participant.

Strategy Adaptiveness Measure

First, all incorrect responses were excluded from analyses. Of the 2107 trials in each condition, 295 (14%) were removed in the choice condition, 263 (12%) in the FA condition and 322 (16%) in the FS condition. Secondly, the *actual change point* was calculated for each participant ($M = 28.08$, $SD = 2.87$). Next, the projected change point was derived from the intersection of both no-choice conditions regression lines.

First, the data before (in FS) and after (in FA) the break point of each regression line were excluded from the data patterns. By using this method, 245 data points were removed from a total of 1844 trials (13%) in the FA condition and 300 of 1785 data points (17%) in the FS condition. Second, by means of the Cook's D statistic

(Myers, 1990; Neter, Kutner, Nachtsheim, & Wasserman, 1996) influential outliers were excluded for each participant. We removed 14 influential outliers on a total of 1599 remaining data points (1%) in the FA condition and 5 of 1485 total data points (<1%) in the FS condition. Third, simple linear regressions were performed on both no-choice conditions and plotted on a single graph. The intersection of both regression lines was the projected change point. The mean *projected change point* was 27.43 ($SD = 1.32$). The mean of the absolute difference between the actual and projected change point, the strategy adaptiveness measure, was 2.12 ($SD = 1.95$).

Correlation and Regression Analyses

First, we calculated the correlations between the strategy adaptiveness measure and the proactive and reactive control measures. The results showed a significant positive correlation between strategy adaptiveness and the reactive measure of the AX-CPT ($r(41) = .36, p = .02$) and strategy adaptiveness and the N-back task's reactive measure ($r(41) = .34, p = .03$). Both correlations indicated that the better the strategy adaptiveness (the smaller the measure), the better the performance on reactive control measures (the smaller the measure). Also, a positive correlation ($r(41) = .39, p = .01$) between Weber fraction and strategy adaptivity was observed. That is, the smaller the Weber fraction (the better the numerosity discrimination), the better the strategy adaptiveness. After performing a Benjamini and Hochberg's (1995) multiple comparisons correction on our data, our effects remained significant. All other correlations were not significant ($p > .09$). In other words, there were no significant correlations between strategy adaptiveness and proactive control, nor between the proactive and reactive control measures (see Table 3).

Table 3

Correlation Matrix

Task	Measure	1	2	3	4	5	6
NJT	1. Strategy Adaptivity ^a	–	.05	.36*	-.02	.34*	.39*
AX-CPT	2. Proactive Control ^b		–	-.01	-.04	.08	-.26
AX-CPT	3. Reactive Control ^c			–	.01	.20	.03
N-BACK	4. Proactive Control ^d				–	.08	-.07
N-BACK	5. Reactive Control ^e					–	.04
PANAMATH	6. Weber Fraction						–

Note. NJT = numerical judgment task. After performing a Benjamini and Hochberg's (1995) multiple comparisons correction on our data, our effects remained significant.

^aStrategy adaptivity was measured by the difference between the actual and projected change point. ^bProactive control in the AX-CPT task was measured by the formula: Errors [BX / (BX + overall)]. ^cReactive control in the AX-CPT task was measured by the formula: Errors [AY / (AY + overall)]. ^dProactive control in the N-back task was measured by the formula: Reaction time [$\text{lure}_{\text{cued}} / (\text{lure}_{\text{cued}} + \text{control}_{\text{cued}})$].

^eReactive control in the N-back task was measured by the formula: Reaction time [$\text{lure}_{\text{uncued}} / (\text{lure}_{\text{uncued}} + \text{control}_{\text{uncued}})$].

* $p < .05$.

As the results showed, magnitude discrimination (Weber fraction) correlated positively with strategy adaptiveness. To exclude the possibility that a good magnitude discrimination had (partly) driven the relation between reactive control and strategy adaptiveness, we calculated partial correlations controlling for individual differences in Weber fraction. The results showed the correlation between reactive control and strategy adaptiveness (AX-CPT: $r(40) = .38, p = .01$; N-back: $r(40) = .35, p = .02$) remained after controlling for Weber fraction as obtained from the magnitude discrimination task. These results showed that individuals who

performed better on the reactive control measures, performed better at selecting the most adaptive strategy (independent of magnitude discrimination).

Because the reactive control measures and Weber fraction did not correlate with each other, which suggested these are three autonomous variables, we were interested to see which model predicts strategy adaptiveness best. Therefore, we have performed a backward selection regression. Backward selection starts from the full model (including all variables) and with every step the least significant variable is dropped until all remaining variables are statistically significant. The results showed that strategy adaptiveness was significantly predicted by including Weber fraction ($\beta = .37$; $p < .01$), reactive control AX-CPT ($\beta = .30$; $p = .03$) and marginally significant by reactive control N-back ($\beta = .27$; $p = .05$; $R^2 = .34$; $F(3,39) = 6.67$, $p < .01$, $\eta_p^2 = .87$). This meant all three variables have a (marginally) significant contribution in predicting strategy adaptiveness.

Discussion

In this study, we have explored how variation in strategy selection is related to individual differences in proactive and reactive control. Individual differences in strategy use might be explained by cognitive control processes needed to select and execute a particular strategy. Therefore, we have investigated the influence of proactive and reactive control by correlating the proactive and reactive measures (of the AX-CPT and N-back task) with the measure of strategy adaptiveness obtained by the numerosity judgment task. The numerosity judgment task (Luwel, Verschaffel, Onghena, & De Corte, 2003) allowed for a clear specification of the strategy repertoire, the strategy that is selected and the adaptiveness of the selected strategy. Our results showed that the measures of reactive control (of both the AX-CPT and the N-back paradigm) correlated positively with strategy adaptiveness. That is, the better the performance of the participant on the reactive control measures, the better strategy adaptivity in the numerosity judgment task.

The present study has shown that cognitive control plays a prominent role in adaptive strategy selection. Our results are in line with previous studies (e.g., Hinault, Badier, Baillet, & Lemaire, 2017; Taillan et al., 2015), which show that cognitive control processes modulate strategy selection. According to Botvinick (2007), conflict monitoring biases behavior towards cognitively efficient tasks and strategies. Even though our task did not contain conflict monitoring, we showed that adaptively selecting strategies can benefit from cognitive control processes and in this case from reactive control processes.

The finding that reactive control, and not proactive control, correlated with strategy adaptiveness suggests that both cognitive control modes might have a different effect on adaptive strategy selection. It seems that adaptive strategy selection benefits from a transient (late) control mode. That is, in the numerosity judgment task, strategy selection happens to be more adaptive upon seeing the stimulus, rather than before the stimulus onset. Also, it seems that adaptive strategy selection would be based more on control mechanisms that are only engaged when needed (i.e., reactive control), which suggests that preparation of the upcoming stimulus characteristics might not be the most efficient way of performing the task. In the present study, it seems that behavior is biased towards efficient strategies in a reactive manner, that is upon seeing the stimulus.

In addition to a significant contribution of reactive control in strategy adaptiveness, we also found that numerosity discrimination is important in predicting adaptivity in the numerosity judgement task. As predicted, the ability to discriminate numerosities correlated with the participant's strategy selection. The results showed

that if a participant performs well in discriminating different magnitudes, the participant selected a strategy in a more adaptive way. We think this finding is a result of the set-up of the numerosity judgment task, rather than a general association between numerosity discrimination and mathematical ability, acknowledging that such association has been debated (e.g., [Schneider et al., 2017](#)). Because the numerosity judgement task requires the discrimination between the number of colored and empty squares to select a strategy, it is logical that numerosity discrimination would also correlate with adaptive strategy selection in such a task. Future research is needed to see if numerosity discrimination would also influence strategy selection in another arithmetic task that is not focused on numerosity discrimination.

For a better understanding of the functional nature of the contribution of reactive control to adaptive strategy selection, it is revealing to look at the difference between the two reactive control measures that we obtained. Indeed, both reactive control measures did not correlate. Assuming enough power, this indicates that both tasks required different aspects of reactive control. In the AX-CPT task, reactive control helped when the cue is uninformative to the probe. If a cue A was shown, the participant had no information about which type of trial this was until the appearance of the probe. On the other hand, the N-back task relied on the fact that the participant needed to wait for the letter to appear to decide. There are two important differences between the control processes required in the N-back task and the processes required in the AX-CPT. First, working memory load was much higher in the N-back task where participants need to keep up to 4 letters in mind and were required to continuously update the string of letters. According to [Braver \(2012\)](#) only proactive control is susceptible to working memory load. Because proactive control requires continuous goal maintenance, resource consumption is high, which results in reduced resources for maintenance of other information needed for the task. In reactive control, on the other hand, cognitive resources are available for other information, because task goals are only transiently activated. Because reactive control cannot account for differences in working memory load, working memory load might not be the main difference between the tasks. Second, the type of control that is needed differed between both tasks. The trials for which reactive control is needed were the AY-trials (AX-CPT task) and the lures (N-back task). This means reactive control in the AX-CPT was more about withholding a prepared response (target-response), while reactive control in the N-back was more about executing the prepared response (non-target response). Our data suggests that both aspects of reactive control are required in strategy selection. This is also shown by the regression analyses, where the best model to predict strategy adaptiveness was the one including both measures of reactive control. This suggests that strategy selection is about finding the right balance between executing and withholding different strategies.

Both proactive control measures did not correlate, which suggests that both tasks might cover another aspect of proactive control. In the AX-CPT task, proactive control was used on trials with an indicative cue. If a cue B was shown, the participant could immediately prepare a non-target response for the upcoming item. However, in the N-back task, the participant could not prepare a response immediately. In the N-back task, the participant had to wait another 2 or 4 items before making a non-target response. Thus, proactive control in the AX-CPT task is more about executing a recently prepared response, while in the N-back task it is about preparing a non-target response, with the possibility that a target-response will be one of the responses in between seeing the cue and executing the response. In general, our results indicated that the AX-CPT and N-back task obtain conceptually somewhat different measures of proactive and reactive control.

A relevant question is why proactive control does not predict strategy adaptiveness. The DMC framework posits that proactive and reactive control might involve independent mechanisms, although this assumption has not

been confirmed directly (Braver, 2012). It is important to note that previous studies have altered the task context (i.e., task demands and task characteristics) to modulate the efficient use of proactive and reactive strategies (Braver, 2012; Braver, Gray, & Burgess, 2007; Bugg, Jacoby, & Chanani, 2011; De Pisapia & Braver, 2006). Typically, a change in proportion of interference alters the control mode selection. In a situation with frequent interference, proactive control might be used more often, because proactive control allows anticipation of interference. In situations with less interference, it seems best to use reactive control. These results show that an alteration on the same task can lead to a change in the individual's preferred cognitive control mode (Braver, 2012; Braver, Gray, & Burgess, 2007; De Pisapia & Braver, 2006). In our task, however, the task characteristics and demands remained the same throughout the experiment, which may have made the need for proactive control redundant.

Additionally, the control mode can also be altered by sequential modulations of arithmetic strategies. Hinault and colleagues (2017) found that strategy selection and execution was more efficient on the current item if the previous item contained interference. This finding was interpreted as reflecting proactive preparation from one trial to the next. In light of this, it is surprising that our results do not show an association of proactive control in strategy selection. This discrepancy between Hinault et al. and the current findings can be interpreted in two ways. First, it could be that the numerosity judgment task involves different control processes and that task characteristics determine which control mode prevails. Hinault et al. used a mental arithmetic task whereas we used a numerosity judgement task, and these task differences could explain differences between Hinault et al. and the current study. Alternatively, the approaches to investigate the influence of cognitive control in strategy selection differ. While we used a correlational approach to investigate the relation of cognitive control and strategy selection, Hinault and colleagues' study looked at sequential effects within a computational estimation task and interpreted their findings as part of the cognitive control processes. Whereas the former method critically depends on the validity of both the cognitive control measures and the arithmetic task, the latter method is crucially based on an interpretation of the trial-to-trial effects. The difference in approach might explain the inconsistencies of the findings. Future research is needed to bring together both methods (correlational approach and interpretation of experimentally induced effects) and see how exactly cognitive control processes are embedded in strategy selection.

Another possible reason for not finding an influence of proactive control, might be that there was limited variation in proactive control. According to the DMC framework, proactive control triggers goal representations before the stimulus is presented. In our task, the goal representations to keep in mind were the strategies that can be used (strategy repertoire) and knowledge on which strategy is best-suited (most adaptive) for which stimulus. Because our task only contained two strategies, we assume that no individual differences emerged because these goal representations were not so cognitively challenging and therefore did not differ substantially between participants. Future research, for example with more cognitively challenging goal representations, might shed some light on the relation of proactive control with strategy adaptiveness.

Even so, it is important to note that the proportion of females and males in the current study is unequal and that females were overrepresented. Clayson and colleagues (2011) have shown gender differences in cognitive control processes. They found that females showed longer reaction times and made more errors compared to males on a Flanker task. As we had a high proportion of females (87% of the sample) in our study, our findings might have been enlarged. On the other hand, not all studies have observed such gender differences on control

measures (Larson, South, & Clayson, 2011), leaving the issue of gender differences unclear. It remains to be determined if gender differences in cognitive control processes are robust and generalize to other tasks.

To conclude, our study was able to isolate the contribution of both reactive and proactive control to adaptive strategy selection, showing that reactive control correlated positively with adaptive strategy selection. Furthermore, two types of reactive control could be distinguished based on the control tasks we used. Individual differences in proactive control did not contribute in the present task. Further research is necessary to determine the exact reasons for the absence of proactive control contributions and for determining whether in other situations proactive control might contribute to strategy selection. Moreover, future research might help to refine the role of reactive control in adaptive strategy selection.

Supplementary Materials

Participant raw data of the present work is available at DANS (Data Archiving and Networked Service): <https://doi.org/10.17026/dans-z22-nq7v>

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Competing Interests

The authors have declared that no competing interests exist.

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